Homework 6 – STAT 543

On campus: Due Friday, March 2 by 5:00 pm (TA’s office);
you also may turn in the assignment in class on the same Friday
Distance students: Due Friday, March 9 by 5:00 pm (TA’s email)

1. Problem 6.24, Casella and Berger (2nd Edition)

2. Problem 7.58, Casella and Berger (2nd Edition) You should recall seeing this distribution on Homework 5.

3. Suppose \( X_1, \ldots, X_n \) are iid normal \( N(\theta, 1) \), \( \theta \in \mathbb{R} \) (i.e., the variance is known to be 1).
   (a) Find the UMVUE of \( \gamma(\theta) = \theta^2 \); call this estimator \( T \). You may assume \( \sum_{i=1}^{n} X_i \) is complete.
   (b) Find \( Var_\theta(T) \). (Try considering a chi-squared distribution for this.)
   (c) Find the CRLB for estimating \( \gamma(\theta) \); call this CRLB_{\theta^2}.
   (d) Show \( Var_\theta(T) > CRLB_{\theta^2} \) for any \( \theta \).

   This is an example where the CRLB isn’t helpful in finding the UMVUE.

4. Problem 7.57, Casella and Berger (2nd Edition) You may assume \( n \geq 3 \).

5. Problem 7.60, Casella and Berger (2nd Edition) and the following:
   (a) Let \( S_n = \sum_{i=1}^{n} X_i \). Using Basu’s theorem, show \( X_1/S_n \) and \( S_n \) are independent.
   (b) Using the result in (a) and \( E_{\beta}(S_n) = n\alpha\beta \), find \( E_{\beta}(X_1/S_n) \).